(1) Consider the three-atom decay sequence \( A \rightarrow B \rightarrow C \), where \( C \) is stable. Defining \( \lambda = 1/T \) where \( T \) is the mean life, and starting with a pure sample of \( N_A^{(0)} \) atoms \( A \), show that the number of atoms \( B \) at a time \( t \) is given by

\[
N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_A^{(0)} \left[ e^{-\lambda_A t} - e^{-\lambda_B t} \right]
\]

Take each of the three limits \( t \ll \{\lambda_A, \lambda_B\} \), \( T_A \ll t \ll T_B \) (for \( T_A \ll T_B \)), and \( T_B \ll t \ll T_A \) (for \( T_B \ll T_A \)), and explain why these are the expected results for \( N_B(t) \).

(2) The graph below on the left, figure 16-13 from your textbook, plots the function \( F(K_{e}^{\text{max}}) \), that is the integral of the beta decay phase space function, in a low \( Z \) nucleus for which the Coulomb correction can be neglected. On the right is shown the beta decay information for \( ^{14}\text{O} \rightarrow ^{14}\text{N} + e^+ + \bar{\nu}_e \), taken from http://www.nndc.bnl.gov/ensdf/.

Using Mathematica or some other program, reproduce Fig.16-13. You’ll need to first deduce the factors and integral for the dimensionless function \( F(K_{e}^{\text{max}}) \). Compare (16-12) and (16-17) but be careful of the words before (16-12); a more mathematically consistent way to write the left hand side of that equation would be \( dR/dp_e \).

Then use this to calculate the three \( \log(FT) \) values shown on the level diagram of \( ^{14}\text{O} \) decay. (The relevant data is included in the level diagram.) Note that the half-life of \( ^{14}\text{O} \) is given in seconds, and branching fraction to individual final states is given by \( I\beta^+ \) in percent.

(3) The following table lists some measurements of a beta spectrum \( R(p_e) \) as a function of electron momentum \( p_e \):

<table>
<thead>
<tr>
<th>( p_e ) (MeV/c)</th>
<th>1.0</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
<th>2.0</th>
<th>2.25</th>
<th>2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R ) (counts)</td>
<td>4500</td>
<td>5750</td>
<td>6310</td>
<td>6380</td>
<td>5670</td>
<td>4630</td>
<td>3100</td>
</tr>
</tbody>
</table>

Make a Kurie plot using this data, and determine the decay energy \( E \).