(1) Consider a triangular function $f(x)$ that forms a straight line from the point at $x = 0$ to the $x$-axis at both $x = \pm \alpha$ and is zero otherwise, that is

![Triangular Function Diagram]

What might you call the “width” $\Delta x$ of $f(x)$? Now determine the Fourier Transform $a(k)$ of $f(x)$, and recommend a value $\Delta k$ for the width of $a(k)$. Show that the product $\Delta x \Delta k$ is consistent with the Uncertainty Principle from quantum mechanics.

(2) A particle with mass $m$ moves under the influence of an attractive $\delta$-function potential energy $V(x) = -aV_0 \delta(x)$ where $a$ and $V_0$ are both positive constants and $a$ has dimensions of length. Find the wave function and binding energy of the ground state.

(3) Consider a particle of mass $m$ subject to a one-dimensional potential of the form

$$V = \begin{cases} 
\frac{1}{2}m\omega^2 x^2 & \text{for } x > 0 \\
\infty & \text{for } x < 0.
\end{cases}$$

Find the energy eigenvalues and wave functions for the ground state and the first two excited states.