Field Mapping of the Temple Polarized $^3$He Target System
v. 1.0

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1 Introduction

The Temple University polarized $^3$He lab utilizes an Helmholtz coil system with radius of approximately 67 cm. The holding field is used to provide the quantization axis in the spin-exchange optical pumping of polarized $^3$He cells. These cells are designed for use in polarized electron scattering experiments at the Thomas Jefferson National Accelerator Facility (JLAB) investigating the spin structure of the neutron.

In this note, we report on the field survey of the Temple system and determine an estimate of the relaxation time due to inhomogeneities of the holding field.

2 Helmholtz Coil Characteristics

The JLAB coils are a double Helmholtz set and the present Temple coils were designed to be an exact duplicate of the smaller JLAB coils. They are an air cooled system designed by Walker Scientific. The specs. of the JLAB system are listed below. Only Axis-2 and Axis-3 are relevant to the Temple system.

<table>
<thead>
<tr>
<th></th>
<th>Axis-1</th>
<th>Axis-2</th>
<th>Axis-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of Uniformity</td>
<td>2400 cm$^3$</td>
<td>2400 cm$^3$</td>
<td>2400 cm$^3$</td>
</tr>
<tr>
<td>Uniformity</td>
<td>&lt;1.0%</td>
<td>&lt;1.0%</td>
<td>&lt;1.0%</td>
</tr>
<tr>
<td>Overall I.D.</td>
<td>56.5&quot;</td>
<td>50.5&quot;</td>
<td>31&quot;</td>
</tr>
<tr>
<td>Overall O.D.</td>
<td>62&quot;</td>
<td>54.75&quot;</td>
<td>33.5&quot;</td>
</tr>
<tr>
<td>Nominal intercoil spacing (face to face)</td>
<td>27&quot;</td>
<td>24&quot;</td>
<td>15.75&quot;</td>
</tr>
<tr>
<td>Resistance (Per coil pair@70F)</td>
<td>3.02Ω</td>
<td>3.05Ω</td>
<td>1.38Ω</td>
</tr>
<tr>
<td>Rated operating current</td>
<td>11.21 A</td>
<td>11.17 A</td>
<td>1.2 A$_{rms}$</td>
</tr>
<tr>
<td>Central field at rated operating current</td>
<td>37 Oe</td>
<td>37.2 Oe</td>
<td>0.106 Oe</td>
</tr>
<tr>
<td>Water cooling</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>AC operation</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Nominal Inductance</td>
<td></td>
<td></td>
<td>112 µH</td>
</tr>
<tr>
<td>Maximum Frequency</td>
<td></td>
<td></td>
<td>100 kHz</td>
</tr>
</tbody>
</table>

3 Measurement Technique

The magnetic field within the Helmholtz system was measured using a Hall probe with 10 mG precision. Procedures to calibrate the probe were followed, although in a gradient measurement only relative field values are of consequence. Forster Magnetoskop$^3$ model 1.068 was also used. Higher precision is possible with this device but it can only tolerate fairly low magnetic fields.
Therefore it was used only for measurements transverse to the main holding field.

In this study, we refer to a right-handed coordinate system with the positive $z$-axis pointing along the field produced by a positive current and with positive $y$-axis pointing vertically upward. To position the probe in the $x$-$z$ plane a grid of holes with 2 centimeter spacing was machined into a horizontal plastic plate and attached to the target lifting mechanism. The origin of the coordinate system was set by positioning a large hole in the center of the grid near the geometric center of the coils. The plate was supported by four posts that made measurements impossible in certain regions. In the data these show up, for example, as gaps centered at ±10 cm in measurements of $B_z(x)$. $y$-dependent measurements were performed using the target lifting mechanism.

Although crude, the horizontal grid provided a fairly reproducible method of positioning the probe. We tested the repeatability of most surveys by making each measurement several times sequentially. In the figures, repeated measurements are signified by different color symbols unless otherwise noted. The spread of these values gives a rough estimate of the error of the measurements.

4 Relaxation due to Field Gradients

Inhomogeneous magnetic fields can cause destruction of the nuclear polarization. This occurs because the atoms precess about the transverse components of the local field produced by the inhomogeneities. The relaxation rate is given by $^2$

$$\Gamma = \frac{1}{\tau} = D \frac{|\nabla B_x|^2 + |\nabla B_y|^2}{B_z^2}$$  \hspace{1cm} (1)

where $D=0.244 \text{cm}^2/\text{s}$ is the $^3\text{He}$ self-diffusion constant, and

$$|\nabla B_i|^2 = \left( \frac{\partial B_i}{\partial x} \right)^2 + \left( \frac{\partial B_i}{\partial y} \right)^2 + \left( \frac{\partial B_i}{\partial z} \right)^2$$

Good cells typically have total lifetimes $\tau$ in excess of 50 hrs. To ensure that the field gradients are not relevant to the total lifetime we require that the lifetime due to magnetic field inhomogeneity is greater than 100 hours. This means that we require gradients to be less than about 35 mG/cm for typical running conditions with a holding field of 26 gauss. A detailed field mapping$^1$ of the JLAB Helmholtz coils was performed before experiment E94010 and all gradients were found to be well within this limit.
Gradient measurements can be confined to two dimensions if we recall that the Maxwell equations $\nabla \times \vec{B} = 0$ imply:

\[
\frac{\partial B_y}{\partial x} = \frac{\partial B_z}{\partial y} \\
\frac{\partial B_y}{\partial y} = \frac{\partial B_z}{\partial z}
\]

and $\nabla \cdot \vec{B} = 0$

\[
\left(\frac{\partial B_y}{\partial y}\right)^2 = \left(\frac{\partial B_z}{\partial x} + \frac{\partial B_z}{\partial z}\right)^2
\]

5 Initial Survey

The initial field mapping of the Helmholtz coils revealed an asymmetry in holding field $B_z$ (See fig. 1). The coils were returned to Walker Scientific and it was determined that one of the coils had an extra winding. After removing the extra winding the coils were returned to Temple and resurveyed.

6 Field Mapping

The field gradients at the target chamber level are shown below. Results are given in mG/cm. The column labeled $I(A)$ represents the current in the coils. Calculated values come from Maxwell's equations.

<table>
<thead>
<tr>
<th>Date</th>
<th>$I(A)$</th>
<th>$\frac{\partial B_x}{\partial x}$</th>
<th>$\frac{\partial B_x}{\partial y}$</th>
<th>$\frac{\partial B_x}{\partial z}$</th>
<th>$\frac{\partial B_y}{\partial x}$</th>
<th>$\frac{\partial B_y}{\partial y}$</th>
<th>$\frac{\partial B_y}{\partial z}$</th>
<th>$\frac{\partial B_z}{\partial x}$</th>
<th>$\frac{\partial B_z}{\partial y}$</th>
<th>$\frac{\partial B_z}{\partial z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>00-23-2000</td>
<td>?</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11-02-2000</td>
<td>?</td>
<td></td>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11-07-2000</td>
<td>?</td>
<td></td>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>32</td>
</tr>
<tr>
<td>11-09-2000</td>
<td>11.00</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>11-21-2000</td>
<td>11.14</td>
<td>4</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>11-22-2000</td>
<td>11.00</td>
<td></td>
<td></td>
<td>4</td>
<td></td>
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<tr>
<td>01-23-2001</td>
<td>11.14</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>01-30-2001</td>
<td>11.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>03-27-2001</td>
<td>11.13</td>
<td>9</td>
<td>4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>03-28-2001(P.C.)</td>
<td>11.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculated</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $\nabla B_x|^2 = 180 | $\nabla B_y|^2 = 1272 | $\nabla B_z|^2 = 2414 |
Note that in order to determine the worst case scenario, the value of the \( \frac{\partial B_z}{\partial y} \) is taken at large values of \( z \), near the edge of a 40 cm target. Values of \( \frac{\partial B_z}{\partial z} \) are an order of magnitude smaller in the central region of the target. Also notice that the only available value of \( \frac{\partial B_y}{\partial y} \) is from June 23, 2000, before the coils were repaired. When there is more than one measurement of a particular gradient, the largest value has been taken. The measurement on 03/28/01 was taken at the pumping chamber level.

At a current of 11.1 Amps the main holding field is about 37 Gauss. We can now evaluate the expected lifetime due to field inhomogeneities:

\[
\tau = 1073h.
\]

The caveats of the previous paragraph, however, indicate that this should only be taken as a lower limit.

7 Conclusion

This study indicates that the relaxation time due to field inhomogeneities is long when compared to other relaxation mechanisms and can probably be neglected. Compared with the JLAB Hall A Helmholtz system, however, the lifetime is a factor of 3 shorter. To understand this difference, it should be realized that the lifetime determined in this study relies on measurements that span almost one year and include data taken before the coils were significantly modified. During the course of that time the alignment of the coils was improved by technician A. Lukhanin so it is probably not reasonable to use the largest values measured for each gradient. In addition, several hardware changes have been made. If the lifetime needs to be determined with greater accuracy, the field mappings should be repeated (preferably all at once or over a short time period) now that the coils are in their final configuration.

Any future survey should be performed at a lower current corresponding to the typical holding field of 26 G and should measure the following gradients at the target chamber level:

- \( (\frac{\partial B_z}{\partial x}) \), \( (\frac{\partial B_z}{\partial y}) \), \( (\frac{\partial B_z}{\partial z}) \)
- \( (\frac{\partial B_y}{\partial x}) \), \( (\frac{\partial B_y}{\partial y}) \)

A more ambitious survey would also include measurements at the pumping chamber level. To ensure that the field is stable, the power supply should be placed in current mode. If voltage mode is used, the coils should be operated at the desired current for several hours prior to the survey in order to avoid temperature dependent drifts in the magnetic field.
8 References

   http://www.jlab.org/e94010/tech_notes.html
3. The technical specs of this device can be downloaded from:
Figure 1: The field survey performed on June 23, 2000. Notice the strongly asymmetric holding field B_z(z)
Figure 2:
Figure 3:
Figure 4: Data (red) compared to the exact solution of Biot-Savart law for $B_z(z)$. Upper curve assumes 16 layers of wire with 16 loops in each layer. Lower curve assumes 16 layers of wire with 15 loops in each layer. The solution that seems to best describe the data has 246 turns of wire. All three curves assume $I=11.136A$ and $R=0.667m$. 
Figure 5:
Figure 6:
Figure 7:
Figure 8:
Figure 9:
Figure 10:
Figure 11:
Figure 12:
Figure 13: Bx(x) measured at the pumping chamber level