Development of the polarized $^3$He target used for measurement of the neutron structure function

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Abstract

We present here a detailed study of the optical line as well as a considerable work on the NMR calibration.
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Chapter 1

General presentation of the experiment

Since 1970, high energy and nuclear physics has been explained with a theory commonly known as the standard model. Later on it became a necessity to test this theory experimentally. QCD, which might explain the behavior of the nucleons, is based on the existence of quarks and gluons. However, none of these particles can be observed directly. So, we have to rely on larger directly observable particles such as the neutron. The study of the spin of the neutron of the polarized $^3$He target will enable us to test the properties of QCD.

1.1 Polarisated $^3$He target

1.1.1 Production of polarized $^3$He samples

The nucleus of a polarized $^3$He atom consists of two spin paired protons and a single unpaired neutron, making it appear approximately as a single polarized neutron. The absence of free neutron targets makes $^3$He a valuable tool in the study of the fundamental structure of the neutron. Thermal agitation makes it very difficult to polarize the sample. To create the polarization, we could use a very strong magnetic field. However, this method is inefficient as shown by this result: a sample placed in a 10 T field at milliKelvin temperatures acquires only about a 4% polarization. A much more reliable way to attain polarization level of 30-50% relies on the process of optical pumping and spin exchange.

Optical pumping relies on the fact that the angular momentum of a photon is transferred to an alkali atom (rubidium in this case). A schematic drawing of the process is shown in figure 1.1. In order to pump only one of the two $5S_{1/2}$ states, we use the angular momentum selection rules in the optical excitation process. When pumping with right circularly polarized light, only the electrons in the $m = -1/2$ state can be excited. However, once excited in $5P_{1/2}$ with
m = +1/2 state, the electrons can decay in both states with equal probabilities. Moreover, the electrons in the m = +1/2 state can no longer be excited, so that we obtain a population of electrons, all with the same magnetic number.

The phenomenon of spin exchange permits the transfer of the polarization of the rubidium to the $^3$He nuclei. This process depends on the hyperfine interaction between the alkalai valence electron and the gas nucleus. This can be expressed schematically as:

$$\text{Rb}(\uparrow) + \text{He}(\downarrow) \rightarrow \text{Rb}(\downarrow) + \text{He}(\uparrow) \quad (1.1)$$

The electron polarization is transferred to the helium nucleus through the hyperfine interaction in collisions. Numerous methods can be used to frustrate depolarizing effects, such as interactions with the container wall.

### 1.1.2 Description of the target

The target (figure 1.2) is filled with rubidium, $^3$He, and nitrogen. Nitrogen is used because it absorbs the photons emitted during the radioactive decay of the electrons of the rubidium from the $5P_{1/2}$ to the $5S_{1/2}$ state. Indeed, these photons have different states of polarization and different spins compared to the photons used during the optical pumping. So, they may limit the efficiency of the pumping effect.

The target is then placed in an oven and heated up to 160°C in order to obtain sufficient rubidium density for optical pumping and spin exchange. Diode lasers and optical devices enable us to create a strong circularly polarized light at the right frequency (see next section and chapter).

In order to check and measure the polarization of the sample, an NMR device is installed around the target. The advantage of the NMR is that it does not destroy the polarization when we take a measurement, thus allowing us to freely
Figure 1.2: Sketch of the target and the process of polarization

check the polarization during the process.
Finally, the whole target is placed in an electron polarized beam which, through
collisions with the $^3$He atom, permits the measurement of the spin of the neutron.

1.2 The optics line

1.2.1 Basis

The optical line is mainly based on the production of circularly polarized light. This characteristic induces strong restrictions since there is only one device which easily transforms the polarization: the waveplate. In particular, a quarter waveplate (QWP) enables the change from linear to circular light and vice-versa, whereas, a half waveplate (HWP) can be used to rotate the plane of the polarized light or turn a right-circularly polarized light into a left-circularly polarized light.

Furthermore, the light emitted by the laser is unpolarized, so our first task is to create a polarized light. In order to accomplish this, we use a polarized beamsplitter since its main use is to split the light into two perpendicular linearly polarized beams. Hence, we obtain two plane polarized lights and we simply have to insert a quarter waveplate in the path of each beam to produce two circularly polarized beams. The two beams are angled so that they cross at the position of the target, i.e., two or three meters beyond the lasers. A simple sketch is shown in the figure below.

![Diagram of the optical line](image)

Figure 1.3: Basic sketch of the optical line composed of a beamsplitter and two quarter waveplates.
1.2.2 The Jones matrices

Before going into more depth on the optical line, we would like to introduce a very useful mathematical representation of the polarization (for more details see [3]). With the help of this method each state of polarization is represented by a two-element matrix which represents the vector $\mathbf{E}$ along the $x$- and $y$-axis of the plane of polarization. For example, a linear light at $+45^\circ$ would be represented by

$$
\mathbf{E} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
$$

The most common Jones vectors are represented on Table 1.1. Moreover, each device which modifies the state of polarization can be represented by a $2 \times 2$ matrix. For our purposes, some matrices of waveplates are given on Table 1.2.

<table>
<thead>
<tr>
<th>General linear polarization</th>
<th>$\mathbf{E} = \begin{bmatrix} \cos \alpha \ \sin \alpha \end{bmatrix}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left circular polarization</td>
<td>$\mathbf{E} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \ i \end{bmatrix}$</td>
</tr>
<tr>
<td>Right circular polarization</td>
<td>$\mathbf{E} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \ -i \end{bmatrix}$</td>
</tr>
</tbody>
</table>

Table 1.1: Summary of Jones vectors

| Quarter Waveplate | slow axis horizontal $e^{-i\pi/4}$ $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$  
                        slow axis vertical $e^{i\pi/4}$ $\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$ |
|-------------------|--------------------------------------------------------------------------------------------|
| Half Waveplate    | slow axis horizontal $e^{-i\pi/2}$ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$  
                        slow axis vertical $e^{i\pi/2}$ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ |

Table 1.2: Summary of Jones matrices
Chapter 2

Detailed study of the optics line

Now that we are more familiar with the whole experiment, we can go into more specific detail. We saw in the last chapter that the polarization of the target is based on the capacity to pump the rubidium. Therefore, we have to develop a design so that we produce a “good” circularly polarized light. This is the focus of this chapter. First, we shall take a look at the general shape of the optical line and derive the choices that we have made. We shall then study the lasers since they are the source of the optical pumping. In conclusion, we shall thoroughly examine the polarization along the line.

2.1 Description

Thanks to the foundations laid in the previous chapter, we can now develop the correct sketch of the optical line and explain the choices we made. Let us take a step by step look at the optical line shown in figure 2.1.

1. The laser and the lens: the beam is transfered from the laser to the beginning of the optical line via the fiber optics. The fiber optics can be oriented more easily than the laser beam. A convex lens is then placed at the right position so that we can observe the image of the tip of the optics fiber on the target. The distance between the end of the optics fiber and the lens is roughly equal to the focal length since the image is nearly at infinity (three or four meters) and the diameter of the fibers is quite small\(^1\). The main reason for using the lens is that even if the divergence of a laser is weak, beyond three or four meters we are always able to see it.

\(^1\)The diameter of fibers does have an effect and we can always observe on the screen the different wires present on the main fiber.
2. The beamsplitter (BS): its main use, as previously explained, is to split the beam light into two components with equal intensity. The light transmitted through the beamsplitter is polarized in the plane of the optical line and is called P component, whereas the reflected light is perpendicular to the plane of the optics line and is called the S component. Since the transmitted light is very good, the reflected light can always be improved, although the beamsplitter is never perfect. Indeed, if we place a polarizer behind each component of the beam and we calculate the polarization (see subsection 2.3.1 for the formula) we find \( Pol = 6\% \) for the transmitted light and \( Pol = 29\% \) for the reflected light. So, we have to construct a convincing design such that both parts of the beam are transmitted through the beamsplitter. The setup is shown in the figure 2.1.

3. The reflected beam: the mechanism to turn the reflected beam into a transmitted beam is shown in the figure 2.2. The reflected beam (P component plus a little S which is the part to remove) is first transformed into a nearly circularly polarized beam via a quarter waveplate tilted to 45° with respect to the P component. After reflection by a mirror, the light goes again through the quarter waveplate so that there is a nearly plane light (but this time we have an S component plus a little P). The S
component of this light is now transmitted through the beamsplitter with a good ratio (we obtain a polarization of roughly 4%) whereas the P component is reflected towards the optics fiber\(^2\). To convince ourselves that the P component is turned into an S component, we can use the Jones matrices, provided we know that the initial state is at 45° of the slow axis of the quarter waveplate. If the initial state of polarization is

\[
E = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

we obtain a final state such that

\[
E = \frac{1}{\sqrt{2}} e^{-i\pi/4} e^{-i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \propto \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}
\]

4. Eventually each beam is transformed into circularly polarized light via two other quarter waveplates tilted with the same angle (+45°) so that we obtain two beams with the same direction of polarization (right or left). In order to change the direction of polarization, two half waveplates can be introduced with a motorized translation stage.

\(^2\)It is important to tilt the beamsplitter a little so that the P component does not go directly into the optics fiber, which could be burned.
2.2 The Diode Lasers

2.2.1 Introduction

The study of the laser is one of the main parts of my work because its characteristics widely influence the efficiency of the pumping. The diode lasers from Optopower Corporation are used because they can produce high power up to 30 W, they are quite cheap in comparison with the other kind of lasers, and they create wide signal in wavelength (which simplifies the pumping at the good wavelength).

The use of a spectrometer enables us to easily plot the spectra of wavelength. For example the study of the process of laser emission at low power can be simply observed as shown in figure 2.3. At low power, the phenomenon of spontaneous emission can be observed and some sharp peaks due to the stimulated emission begin to appear.\(^3\) At very low power, we have no population inversion. Only the spontaneous emission is present: we can monitor a spectra roughly similar to a lorentzian (cf figure 2.4).

\(^3\)We can observe many peaks for the laser emission because the lasers used are multimode (characteristics related to the optical cavity).

![Figure 2.3: Stimulated and spontaneous emission at low current. Observation of the multimode emission.](image-url)
2.2.2 Light versus current and wavelength versus current curves

The curves presented in this section are very important since they enable us to calculate the efficiency of different lasers and to calibrate them so that they emit light at a good wavelength. The wavelength and the optical power of a laser depend on two main parameters: the drive current and the temperature inside the cavity.

The optical power versus drive current curve is shown in figure 2.5. This curve permits us to measure the degree to which a diode laser emits light as current is injected into the device. As the injected current is increased, the laser, first demonstrates spontaneous emission, which grows very slowly until it begins to emit stimulated radiation at the threshold current (roughly at 6 A).

A laser diode, which has a good rate of converting the input electric power as output light power, is obviously a device that is performing well. The slopes of the light versus current curves serve as a direct measurement of the ability of the device to exact. As can be seen in figure 2.5, laser 135, which is a 30 W laser, is working well, 15 W laser is naturally half of the efficiency of the previous one, and laser 129, which should be a laser of 30 W, looks damaged.

The optical power versus drive current curve is the first graph to trace when we want to diagnose a laser. However, some other curves, like the wavelength versus injected current, are very interesting. In order to emit light with the correct wavelength (795 nm in our case), we have to understand the behavior of the wavelength of each laser with respect to the current and the temperature. The correlation between the wavelength and the current is very simple, since as shown in figure 2.6, the curves are linear. These curves are plotted by a spectrometer which represents the spectra of the wavelength at each injected current. We then calculate the mean value and the root-mean-square of each spectra.
We can observe that if laser 135 reaches 795 nm at about 25 A, the 15 W laser is unable to do so. Even the root-mean-square does not include 795 nm. So, we notice that the 15 W laser is useless for our experiment since its spectra does not include 795 nm and cannot pump the rubidium. Different spectra for a given intensity are shown in figure 2.7. We notice the gap between the spectra of the 15 W laser and the others. We also noticed a rather unusual behavior of the spectra of laser 129, which is damaged.

2.3 Polarization of the light

2.3.1 Presentation and theory

The study of the polarization, especially at the position of the target, is important since the efficiency of the pumping depends upon the quality of the circularly polarized light. We could have very powerful lasers to increase the efficiency of the pumping. However, if half of the power is useless because the light is elliptical, then the result is the same as if we had half power lasers with good settings. Since each diode laser is very expensive, we can easily understand the interest of studying the polarization.

First of all we have to find a formula which expresses the quality of the polar-
**Figure 2.6:** Wavelength versus current curves for different lasers.

**Figure 2.7:** Spectra of the wavelength at a given current.
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...ization. We, of course, would like to obtain 100% polarization when the light is circularly polarized and 0% polarization when we have a plane polarized light. We begin by writing the electric field vector in the plane of polarization (cf figure 2.8)

\[ \mathbf{E} = x \mathbf{i} + y \mathbf{j} \]  \hspace{1cm} (2.1)

\[ x = A \cdot \cos(kz - \omega t) \]  \hspace{1cm} (2.2)

\[ y = A \cdot \cos(kz - \omega t + \epsilon) \]  \hspace{1cm} (2.3)

where \( \epsilon \) measures the degree of polarization. Indeed, if \( \epsilon = 1 \), we have a linearly polarized light, and if \( \epsilon = \frac{\pi}{2} \), we obtain a circularly polarized light.

If we expand both equations, we have

\[ \frac{y}{A} = \cos(kz - \omega t) \cdot \cos(\epsilon) - \sin(kz - \omega t) \cdot \sin(\epsilon) \]  \hspace{1cm} (2.4)

\[ \frac{x}{A} = \cos(kz - \omega t) \]  \hspace{1cm} (2.5)

We square (2.4) and (2.5) and rearrange the terms. After a bit of algebra we get:

\[ \frac{x^2}{A^2} + \frac{y^2}{A^2} - \frac{2xy \cos \epsilon}{A^2} = \sin^2 \epsilon \]  \hspace{1cm} (2.6)

which is the equation of an ellipse.

Furthermore, we have \( A^2 = \frac{1}{2}(a^2 + b^2) \), where \( a \) and \( b \) are the semimajor and semiminor axes respectively. There are a couple of pages of algebra involved, but it works out nicely.

When a linear polarizer is rotated in this light, the maximum and minimum
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intensity transmitted are respectively $I_{\text{max}} = a^2$ and $I_{\text{min}} = b^2$. Following the same reasoning and manipulations we find:

$$\sin \epsilon = \frac{ab}{A^2}$$  \hspace{1cm} (2.7)

Setting the polarization $Pol = \sin \epsilon$ we have:

$$Pol = \frac{ab}{\frac{1}{2}(a^2 + b^2)} = \frac{2\sqrt{I_{\text{max}} \cdot I_{\text{min}}}}{I_{\text{max}} + I_{\text{min}}}$$ \hspace{1cm} (2.8)

$$Pol = \frac{2\sqrt{I_{\text{max}}}}{1 + \frac{I_{\text{min}}}{I_{\text{max}}}}$$ \hspace{1cm} (2.9)

Equation (2.9) is represented in figure 2.9.

![Figure 2.9: Polarization versus $\frac{I_{\text{min}}}{I_{\text{max}}}$ curve. Moreover, we can see the characteristic values for different parts of the beam of the beamsplitter (BS) and for different quarter waveplates (QWP).](image)

2.3.2 Results

Since we have just defined a formula capable of putting a figure to the polarization, we can now use it to study the polarization for different positions in
the optical line. It enables us to measure and evaluate the influence of different devices on the polarization.
We can describe quickly how to perform this measurement.

**Principle of the measurement**

The measurement in fact, is relatively simple and use only two devices: a polarizer of good quality (at least better than the devices we have to analyze) and a powermeter. The polarizer enables us to select a direction of polarization whereas the powermeter measures the intensity along this direction. When we steadily turn the axis of the polarizer, we observe that the intensity changes periodically (excluding the circularly polarized light). Indeed, if we have an elliptically polarized light, the intensity ranges between two values corresponding to the major and minor axes of the ellipse (cf figure 2.8). If the difference between the minimum and maximum values is sharp, we can then conclude that we have a nearly plane polarized light. On the other hand, if the difference is weak and we obtain a horizontal line, we have a nearly circularly polarized light.

In summary, we steadily turn the axis of the polarizer with a motor during many periods, and we simultaneously measure with a powermeter the intensity that we plot on LabView. An example of acquisition is presented in figure 2.10.

![Graph](image)

**Figure 2.10:** Acquisition on LabView of the optical power during many rotations of the polarizer. From here we can extract the maximum and minimum intensities.

**Beamsplitter and quarter waveplates.**

We shall come back to section 2.1.3 since the polarization of the reflected beam behind the beamsplitter is worse than the polarization of the transmitted beam. We can easily install our device (polarizer and powermeter) behind each beam. First, when we put it just behind the reflected beam, we find a polarization of roughly 29%. Then, when we set it up behind the transmitted beam, we find a polarization of about 6.5%. Finally, we install our device behind the beam,
CHAPTER 2. DETAILED STUDY OF THE OPTICS LINE

which was first reflected, then cleaned and transmitted (cf figure 2.2). After doing so we obtain a polarization value of 5.3 %.
Furthermore, we can put our device behind the end of the optics line and measure the efficiency of the circularly polarized light. But this efficiency depends upon the settings and the quality of the last quarter waveplate in the beam (since that is the one which produces the circularly polarized light). We can thus test the quality of the different quarter waveplates we are using. As shown on figure 2.9, one of these quarter waveplates is very efficient \((Pol = 99.91 \%)\) whereas the other two are not as efficient \((Pol = 98.02 \% \text{ and } Pol = 97.37 \%)\). However, these results depend on the settings which may vary considerably between two experiments.

**Polarization at the position of the target**

Finally the study of the polarization at the position of the target is important, since we didn’t know what was the influence of the sphere which is filled with the gas, on the polarization. The results are presented in figures 2.11 and 2.12. Figure 2.11 represents the optical power in function of the position along the \(x\)-axis at a given height. Every millimeters we took a measurement analog as the one shown in figure 2.10 and we could thus calculate the polarization at this position (figure 2.12). Without the target, the numerous peaks are due to the fact that the main fiber is constituted of many wires, so that each wire creates its own image. The polarization in the absence of target is roughly independent of the position. When a target of spherical shape cross the beam, the situation is completely different. The beam is a little larger than the target so that we can observe it near 2.5 cm and 6 cm. Then the presence of the target alter the intensity and the polarization in a way proportional to the angle between the beam the surface of the target (spherical). However the intensity without the target along the \(x\)-axis is not homogeneous, as shown on figure 2.11. Thus, if we try to calculate the polarization in function of the angle with a flat glass that we steadily angle, we don’t find exactly the same result that with the target. But we still observe the sharp decrease of the polarization when the angle between the surface and the beam is important. We can conclude that the presence of the target alter the polarization, but only at the edges.
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Figure 2.11: Optical power versus position along the x-axis. The curve of the beam is represented without the target, in dashed line and with the target, in solid line.

Figure 2.12: Polarization versus position along the x-axis
Chapter 3

NMR Polarimetry

As we saw in the first chapter, the polarization of the sample can be measured by two different techniques. One method is the NMR using Adiabatic Fast Passage (AFP), the other is a rather novel technique relying on the shift of the Rubidium Zeeman Resonance (EPR) frequency due to $^3$He polarization. For our purpose, we shall only focus on one of these methods, the NMR, and more particularly, on the water calibration. We shall begin by introducing some of the fundamental concepts relating to the NMR, then we shall examine the way we perform the measurement and eventually present some results.

3.1 Fundamental concepts

The main device is based on three coupled coils: the Helmholtz coils which create a magnetic moment, the RF coils which introduce a spin flip of the magnetic moment, and the pick-up coils which detect the signal.

3.1.1 The holding field $B$

The molecules in the water sample have non-zero magnetic moment carried by the protons of the hydrogen nuclei. When placed in a steady magnetic field $\mathbf{B}$ (the holding field) these protons tend to align along this field (that we choose along the $z$-axis) in one of two possible orientations. In fact, each spin (or magnetic moment) precesses about the $z$-axis with the Larmor frequency $\gamma B$. If we try to quantify the magnetic moment, we have to use a statistical mixture of the two orientations corresponding to the state vectors $|+>$ and $|->$ (see for example [1]).

We have indeed:

$$\rho = \sum_\epsilon p_\epsilon \cdot \rho_\epsilon$$  \hspace{1cm} (3.1)

with

$$\rho_\epsilon = |\epsilon><\epsilon|$$

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\[ \epsilon = +or- \]

and

\[ p_e = \frac{1}{Z} e^{-\frac{\epsilon \omega_0}{kT}} \]
\[ Z = e^{-\frac{\omega_0}{kT}} + e^{+\frac{\omega_0}{kT}} \]
\[ \omega_0 = \gamma B \]

We thus obtain:

\[ \rho = \frac{1}{Z} \begin{pmatrix} e^{-\frac{\omega_0}{kT}} & 0 \\ 0 & e^{+\frac{\omega_0}{kT}} \end{pmatrix} \] (3.2)

Hence:

\[ < S_x > = Tr(\rho S_x) = 0 \] (3.3)
\[ < S_y > = Tr(\rho S_y) = 0 \] (3.4)
\[ < S_z > = Tr(\rho S_z) = -\frac{\hbar}{2} \tanh \left( \frac{\hbar \omega_0}{2kT} \right) \] (3.5)

Finally,

\[ < M_z > = \gamma < S_z > = \chi B \] (3.6)

with

\[ \chi = \frac{\hbar \gamma}{2B} \tanh \left( \frac{\hbar \gamma B}{2kT} \right) \]

The signal detected is proportional to the polarization of the sample. We thus see that if we calculate \( \chi \) exactly, especially for a weak field, where \( \chi \) is independent of \( B \), we can check that our signal satisfies the equation and we can calibrate \( B \) as a function of it.

3.1.2 Magnetic resonance

As explained in the first part of this section, if a magnetic moment is immersed in a steady field \( B \) in the \( z \) direction, the moment will precess about the \( z \)-axis with the Larmor frequency \( \gamma B \). The measurement of the magnetic moment is made through inducing a spin flip of the magnetic moment. As in the corresponding classical motion, in order to change the angle that makes the magnetic moment with the \( z \)-axis, it is necessary to impose an additional transverse magnetic field normal to the plane through the \( z \)-axis and the magnetic moment (cf figure 3.1 and [2]). Since this plane rotates with the Larmor frequency, we may expect a spin flip of the magnetic moment to be induced when the frequency \( \omega \) of an imposed transverse magnetic field \( B_1 \) along the \( x \)-axis for example, is equal to the Larmor precessional frequency.

Hence, we obtain the important relation:

\[ \omega = \gamma B \] (3.7)
Figure 3.1: Transverse field $B_1$ imposes a torque $N$ that cause $m$ to change its orientation with respect to the $z$ axis.

clean.eps This spin flip induces the change in the magnetic flux along the $y$-axis.\footnote{Only the spin flip creates a magnetic flux along the $y$-axis since the other fields are perpendicular here.} A third couple of coils, called the pick-up coils, are installed along the $y$-axis so that the variations of magnetic flux are detected by the creation of a current inside the coils. We thus obtain our signal proportional to the polarization of the sample.

### 3.2 Description of the device and results

#### 3.2.1 Adiabatic Fast Passage

We saw in the previous section that when equation (3.7) is satisfied, a magnetic flux through the pick-up coils is created. However, there are two means to satisfy this equation. The first task is to set a given homogeneous magnetic field $B$ and vary $\omega$ till we obtain the resonance. On the other hand, the counter process is also possible, i.e. set $\omega$ and vary $B$. In our case, we used the latter per facility. First, we need to create a homogeneous magnetic field (about 34 Gauss) with the Helmholtz coils so that the value of the magnetic field may vary. In order to do this, we use a function generator commanded by Labview and a Kepco power supply which serve as amplifier. In the same time we apply an RF current with another function generator to the RF coils at a given frequency (often 91 kHz).
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Usually the transverse magnetic field created is roughly about 100 mG. Finally, we sweep up and down the magnetic field \( B \) near the resonance by sending a linear ramp from LabView to the power supply and measure the current induced through the pick-up coils. However, care needs to be taken when performing the experiment. First, the sweep has to be slow enough so that the spins can follow the changing field adiabatically. This condition can be expressed by the relation:

\[
\frac{\dot{B}}{B_1} \ll \omega = 5.7 \times 10^6
\]  

(3.8)

The sweep must not be too slow because of the spin relaxation, namely:

\[
\frac{\dot{B}}{B_1} \geq \frac{1}{T_1} \approx 2.3 \times 10^{-3}
\]  

(3.9)

Typically, these two relations lead to \( \dot{B} = 1.2 \) G/s. The next concern is the difficulty in detecting the weak signal from water. In order to resolve this, we need to amplify the signal. However, when we amplify the signal the background noise is increased by the same factor. Therefore, we use a Lock-in amplifier to select a single frequency in the signal.

The main process is as follows: The signal used by the RF coils is sent to the lock-in amplifier as a reference. The signal detected by the pick-up coils has the same frequency as the one sent to the RF coils, since the spins flip at the RF frequency (cf figure 3.2). The signal delivered from the pick-up coils is then multiplied by the reference signal. When integrated over time, only the obtained frequency equal to the frequency given by the reference, is different from zero (Fourier decomposition). Thus, we can eventually obtain a signal.

![Diagram](image)

Figure 3.2: Description of the lock-in amplifier.
3.2.2 Results

Though we use numerous devices, the water signal is still very difficult to observe. We need to place exactly the pick-up coils and the sample at the right position to minimize the noise. Then we need to do the experiment at a quiet moment of the day, since many effects like electronic devices, vibrations, lamps, air flow, can be detected. Finally, even with these cares, many sweeps are necessary, so that we cancel the random noises (typically between 20 and 200 sweeps according to the quality of the settings). An example of water NMR signal is presented in figure 3.3. In this case, we did twenty sweeps, but the quality is poor and the shape of the peak is not clear. Once we obtain a nice water signal (unfortunately we did not), we can fit it and determine with accuracy the polarization and the value of the RF field. The fit is not only lorentzian, but based on the Bloch equations such as very good resolution can be obtained. The observation of the He\(^3\) signal, contrary to the water signal, is easier to observe (less thermal relaxation effects), but the principle of the measurement stay the same. In conclusion, the NMR is very useful, since it is almost non-destructive and enables accurate measurements of the polarization.

![Graph](image.png)

Figure 3.3: Example of water signal effectued after twenty sweeps.
Bibliography