The extraordinary advances in quantum control of matter and light have been transformative for precision measurements enabling probes of the most basic laws of Nature to gain a fundamental understanding of the physical Universe. The long-lived isomer in $^{229}$Th, first studied in the 1970s as an exotic feature in nuclear physics, is the only known candidate for the development of a nuclear clock. The transition energy between the ground and first excited states of $^{229}$Th is unusually small and amounts to only several eV, making it the only laser-accessible nuclear transition. An optical clock based on this transition is expected to be a very sensitive probe for variation of fundamental constants, searches for violations of Einstein's equivalence principle, and ultralight dark matter. I will discuss these opportunities to discover new physics with a nuclear clock on the ground and in space.