Physics 5501: Mathematical Physics

Fall 2014

106 Barton Hall A; Monday, Wednesday, 1:00 - 2:20 p.m.

Instructor

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Course objectives

The course is designed to provide the student with the basic concepts and working knowledge in the modern methods of mathematical physics, at a graduate level. The student is expected (i) to be able to analyze mathematical aspects of physical models and uncover inherent difficulties; (ii) to be able to conduct qualitative analysis preceding computer simulations; (iii) to acquire essential knowledge, overview, and expertise for reading current research literature.

Textbook

E. Butkov, *Mathematical Physics*, Addison-Wesley

Additional Reading

Introductory Reference
M.L. Boas, *Mathematical methods in the physical sciences*, 3rd ed., Wiley, 2006.
S.M. Lea, *Mathematics for physicists*, Brooks/Cole, 2004.
J. Mathews and R.L. Walker, *Mathematical methods of physics*, Benjamin, 1970.

Advanced
G.B. Arfken and H.J. Weber, *Mathematical methods for physicists*, Elsevier, 2005
R. Courant and D. Hilbert, *Methods of Mathematical Physics* Wiley, 1989
V.V. Mitin, D.A. Romanov, and M.P. Polis, *Modern advanced mathematics*, Wiley, 2001
P. Szekeres, *A course in modern mathematical physics*, Cambridge University Press, 2004
E.W. Byron and R.W. Fuller, *Mathematics of classical and quantum physics*, Dover, 1992

Class Procedures

Class will meet for lectures two days per week at the scheduled time. Class attendance is governed by University Policy; however, each student is responsible for all materials covered in all lecture sessions. (This means calling someone in the class before the next classtime to obtain missed material, handouts and schedule information.)

Assignments and exams

Weekly assignments of homework problems worth a total of 50% of the course grade will be given (late homework returns will incur 1 point penalty for each week of delay). Reading assignments will be made to assist in material assimilation. A midterm exam (**October 15**; 25%) and a final exam (**December 15**; 25%) will be given as in-class closed-book exams.

Grade scale is as follows: greater than 85% – A, 82-85% – A-; 78-81% – B+, 74-77% – B, 70-73% – B-; 66-69 – C+, 62-65% – C, 58-61% – C-; you guess what happens below 60%.

There will be absolutely **NO make-up exams administered for ANY reason.** If any student finds it necessary to miss an exam due to reasons of health or other extraordinary reasons, it is the responsibility of the student to inform the instructor **in writing, prior to the exam,** of these circumstances (the student should be prepared to completely justify the reason for the absence). If the reason for the absence is approved, the appropriate course of action will be decided by the instructor on a case-by-case basis. Any student missing a scheduled exam without contacting the instructor will be assigned a grade of 0 (zero) for that exam.

Handicapped students

Any student who has a need for accommodation based on the impact of a disability should contact me privately to discuss the specific situation as soon as possible. Contact Disability Resources and Services at 215-204-1280 in 100 Ritter Annex to coordinate reasonable accommodations for students with documented disabilities.

Course Topics

1. Complex Variables
   a. Algebra and geometry of complex numbers. Powers and roots. Complex functions. Euler's formula and its applications. Multivalued functions
   b. Analytic functions. Integral theorems. Cauchy integral formula
   c. Complex sequences and series. Taylor and Laurent series
   d. Zeros and singularities. The residue theorem and its applications
   e. Bonus: the saddle-point method
2. Fourier series from complex standpoint
3. Fourier Transform and Distributions
   a. From Fourier series to Fourier transform
   b. Properties of Fourier transform. Fourier integral theorem
   c. Dirac delta-function and its representations. Delta-sequences
   d. Distributions and weak convergence
   e. Properties of distributions
   f. Fourier transforms of distributions
4. Laplace Transform
   a. From Fourier transform to Laplace transform
   b. Mellin inversion integral
   c. Basic properties of direct and inverse Laplace transform
   d. Operational calculus
   e. The convolution theorem and additional properties of the Laplace transform
   f. Application of Laplace transforms

5. Ordinary differential equations revisited
6. Partial Differential Equations
    a. Wave equation. Classification of second-order equations
    b. The method of separation of variables
    c. Laplace and Poison equations
    d. The diffusion equation. The Schrödinger equation. The Helmholtz equation
    e. Use of Fourier and Laplace transforms
    f. The method of eigenfunction expansions and finite transforms
    g. Continuous eigenvalue spectrum
7. Series of special functions
    a. Cylindrical and spherical coordinates
    b. The common boundary-value problem
    c. The Sturm-Liouville problem. Self-adjoint operators
    d. Legendre polynomials
    e. Fourier-Legendre series
8. Finite-dimensional linear spaces
    a. Normal coordinates and linear transformations in classical mechanics
    b. Vector spaces, bases, and coordinates
    c. Linear operators, matrices, inverses
    d. Changes of basis
    e. Inner product. Orthogonality. Unitary operators
    f. Eigenvalue problems. Diagonalization.
9. Infinite-dimensional vector spaces
    a. Linear spaces of functions
    b. Banach and Hilbert spaces
    c. The harmonic oscillator in quantum mechanics
    d. Matrix representations of linear operators
    e. Algebraic methods of solution
    f. The Hemite polynomials
10. Green's functions
    a. Green's function for the Sturm-Liouville operator
    b. Series expansions for Green's functions
    c. Green's functions in two dimensions
    d. A case of continuous spectrum
    e. Applications of Fourier transform. The principle of causality
11. Groups and their representations
    a. Definitions and examples. Abelian and Non-Abelian groups
    b. Subgroups
    c. Group representations
    d. Reducible and irreducible representations. Characters
    e. Physical applications of group theory
12. Probability and statistics revisited

The course content may be modified at the discretion of the instructor. The instructor will announce in class any material so affected.