The modern theories directed toward unifying gravitation with the three other fundamental interactions suggest variation of the fundamental constants. While the energy scale of such physics beyond the Standard Model is much higher than that currently attainable by particle accelerators, the variation of the fundamental constants may nevertheless be detectable via precision measurements at low energies. I will give an overview of various searches for new physics with atomic clocks and focus on proposals for future experiments, in particularly with highly-charged ions. These experiments are also directly relevant to ultra-light dark matter searches.