NEW TESTS FOR AND BOUNDS ON NEUTRINO MASSES AND LEPTON MIXING

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We propose a new class of correlated tests for neutrino masses and lepton mixing. Two particular tests based on $\pi$, $K\ell_2$ decay and nuclear $\beta$ decay are discussed and applied to present data to derive bounds on these quantities.

The lepton mass spectrum remains a puzzle at present. In particular, the question of possible neutrino masses and associated nonzero leptonic mixing angles is still an open one. In this letter we present a new class of correlated tests for neutrino masses and lepton mixing angles. Two of the most promising tests make use of (1) $\pi$ and K leptonic decay; and (2) nuclear $\beta$ decay. They can detect neutrino masses in the ranges $\approx 1$ to $\approx 400$ MeV and $\approx 0.1$ keV to $\approx 5$ MeV, respectively, and, especially in the case of the first test, are sensitive to very small mixing angles.*1 The second test can be applied to existing data; however, an optimal application of the first test requires a new (but not overly difficult) experiment, which we propose. There could easily be a massive neutrino signal which would not have been detected in any previous experiment but would be observed by the first test.

The observation underlying this new class of tests is that in previous direct searches, i.e. via decays, for neutrino masses it was implicitly assumed that in a decay of the form $X \rightarrow Y + \ell_a + (\bar{\nu})_{\ell_a}$, where $\ell_a = e$ or $\mu$, $X$ denotes the parent particle, and $Y$ denotes a possibly null set of final state particles, the $(\bar{\nu})_{\ell_a}$ is a definite particle with mass $m(\nu_{\ell_a})$, which could be nonzero and on which the experiment would set an upper limit. In the context of the present gauge theory of electroweak interactions this is equivalent to assuming that the possibility of defining the neutrino weak gauge group eigenstates to be simultaneously mass eigenstates obtains in the case of massive (nondegenerate), as well as massless, neutrinos. However, this assumption is not in general valid; in the former case the weak eigenstates $\nu_a$, where $\ell_a = \{\ell_1 = e, \ell_2 = \mu, \ell_3 = \tau, \ldots, \ell_n\}$ are linear combinations of the mass eigenstates $\nu_i$, $i = 1, \ldots, n$. In the standard $SU(2)_L \times U(1)$ electroweak theory this mixing is specified by the unitary lepton mixing matrix $[1] ^2 U: \nu_{\ell_a} = \sum_{i=1}^{n} U_{ai} \nu_i$. Thus, for example, a decay of the form $X \rightarrow Y + \ell_a + \bar{\nu}_{\ell_a}$ would actually consist of an incoherent sum of the separate modes $X \rightarrow Y + \ell_a + \bar{\nu}_i$, where $i$ runs over the subset of the $n$ neutrino mass eigenstates allowed by phase space. The branching ratio for the $i$th mode is modulated by the mixing matrix factor $|U_{ai}|^2$ as well as a kinematic factor depending on $m(\nu_i)$. An obvious generalization of this comment applies to the pure leptonic decay of the form $\ell_a \rightarrow \nu_{\ell_a} \ell_b \bar{\nu}_{\ell_b}$ which in general consists of all of the modes $\ell_a \rightarrow \nu_i \ell_b \bar{\nu}_j$ kinematically allowed, each with $U$-dependence $|U_{ai} U_{bj}|^2$.

Let the $i$'s which label the $\nu_i$ be divided into light

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*1 Some grand unified models feature nonzero neutrino masses; however, these tend to be in the range $10^{-5} - 10$ eV and hence would not be detected by our test. We stress that the main interest of our test is to search, without any theoretical predilections, for $m(\nu_i)$ in a sensitive way not hitherto exploited.

*2 The mass eigenbasis is ordered such that $|U_{aa}|$ (no sum on $a$) $> U_{a1}$, $a \neq i$. This of course does not imply that $m(\nu_i) > m(\nu_j)$ if $i > j$, although it would be natural to assume this inequality, given the monotonic generation dependence of quark masses.
and heavy subsets \{i_L\} and \{i_H\} such that \(i \in \{i_L\}\) if \(m(\nu_i) \lesssim 0.1\) keV and \(i \in \{i_H\}\) otherwise. For a given decay such as \(X \rightarrow Y + \ell_a + \nu_i\), we define the light dominantly and subdominantly coupled (LDC and LSC) modes as those for which \(i \in \{i_L\}\) and \(|U_{ai}|^2\) is of order unity or \(|U_{ai}|^2 \ll 1\), respectively. The heavy dominantly and subdominantly coupled (HDC and HSC) modes are similarly defined but with \(i \in \{i_H\}\), as allowed by phase space. For \(i \in \{i_L\}\) the constraints on leptonic mixing do not necessarily prevent some \(|U_{ai}|^2\), \(a \neq i\), from being comparable to \(|U_{aa}|^2\) (no sum on \(a\)); i.e. there may be several LDC modes. One may distinguish three cases for \(i \in \{i_L\}\) (no sums on \(a\)): (1) \(|U_{ai}|^2 \ll |U_{aa}|^2\) for all \(i \neq a\), i.e. one LDC mode and the rest LSC modes; (2) \(|U_{ai}|^2 \lesssim |U_{aa}|^2\) for some \(i\) and \(|U_{ai}|^2 \ll |U_{aa}|^2\) for the other \(i\), i.e. several LDC and LSC modes; and (3) \(|U_{ai}|^2 \lesssim |U_{aa}|^2\) for all \(i\), i.e. all LDC and no LSC modes. For \(i \in \{i_H\}\) the various constraints do imply that there is only one HDC mode, namely \(i = a\).

Thus, the mass limits \(m(\nu_e) < c^2_a\) (90\% CL), \(a = e\) or \(\mu\), quoted in past experiments must really be reinterpreted as the statements \(m(\nu_i) < c^2_a\) for all LDC modes \(i\) in the relevant decay. (The statistical errors would also increase for each of the \(i\) bounds if there were several.) These limits do not constrain the LSC or HSC modes. Our tests are designed to detect the HSC modes.

The first and cleanest test based on these observations makes use of the leptonic decays \(M^+ \rightarrow \ell^+_a + \nu_a\) where \(M = \pi\) or \(K\) and \(\ell_a = e\) or \(\mu\). The decay \(\pi^+ \rightarrow \mu^+ + \nu\) was used in a recent Swiss Institute for Nuclear Research (SIN) experiment [2] to set the best upper bound on \(m(\nu_\mu)\), quoted as \(m(\nu_\mu) < 0.57\) MeV (90\% CL). It was assumed that the muon momentum spectrum would consist of a single monochromatic line \(^{\text{a}}\) at

\[|p_\mu| = \lambda^{1/2}(m_\mu^2, m_\mu^2, m(\nu_\mu)^2)/(2m_\mu),\]

where

\[\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx);\]

hence a measurement of \(|p_\mu|\) would yield an upper bound on \(m(\nu_\mu)\). A similar comment applies to the earlier \(\pi_e\) experiments, such as the one by the Columbia group [3] \(^{\text{a}}\) to experiments on \(K_{e2}\), of which the most recent was by a CERN–Heidelberg collaboration [4]. In actuality, however, if neutrinos are massive (and nondegenerate), the \(\ell_a\) momentum spectrum in the decay \(M^+ \rightarrow \ell_a^+ + \nu_a\) would in general consist of \(k\) monochromatic lines \(^{\text{a}}\) at

\[|p_a| = \lambda^{1/2}(m_M^2, m_a^2, m(\nu_i)^2)/(2m_M)!\]

To our knowledge, no experiment has specifically searched for this clear signature of massive neutrinos, and we propose such a search. Similarly, \(M^+ \rightarrow e^+ \nu_e\) experiments (e.g. ref. [3]) which measure directly the electron energy rather than its momentum should search for the \(k\) lines which would be expected at

\[E(\ell_e) = [m_M^2 + m_e^2 - m(\nu_i)^2] / (2m_M).\]

Let us describe this test further. First, it has the advantage of being purely kinematic, independent of whether other exotic effects, such as very weakly coupled currents with different Lorentz structure than \(V - A\), or flavor changing Higgs bosons are present. Secondly, if a signal is observed, one can immediately determine independently and unambiguously for each line, \(|p_a|\) the corresponding \(m(\nu_i)\). A very important merit of the test is that this mass determination is independent of mixing angles, except in the minimal sense that \(|U_{ai}|\) must be large enough for a signal to be detected in a given experiment \(^{\text{a}}\). Next, knowing the mass \(m(\nu_i)\) and, at this point, assuming that the relevant couplings are \(V - A\), one can use simple kinematics to determine

\[R_{ai} = |U_{ai}|^2 \left| \left( \sum_{\text{LDC}} |U_{ai}|^2 \right) \right| \approx |U_{ai}|^2,\]

\(^{\text{a}}\) A similar \(\pi_e\) experiment by D. Bryman et al. is currently in progress at TRIUMF.

\(^{\text{a}}\) This should be contrasted with another test for HSC modes, viz. the ratio \(B(M^+ \rightarrow e^+ \nu_e)/B(M^+ \rightarrow \mu^+ \nu_\mu)\), and with the \(\nu\) oscillation test for lepton mixing. The former only measures an integrated effect due to all HSC modes present and is incapable of determining \(m(\nu_i)\) or \(|U_{ai}|\) for any particular mode. The latter can only yield correlatd information on the quantities \(m(\nu_j)^2 - m(\nu_i)^2\) and \(U_{ai}\), never the set of \(m(\nu_i)^2\) in isolation, much less a particular \(m(\nu_i)\). On the other hand, it is sensitive to values of \(m(\nu_i)\) far below the level detectable by our tests.
for each of the HSC lines $i \pm 6$. The rate for the mode $M^+ \to \xi^+_a \nu_i$ relative to that for the mode $M^+ \to \xi^+_a \nu_a$ with $m(\nu_a) = 0$, is given by

$$\frac{\Gamma(M^+ \to \xi^+_a \nu_i)}{\Gamma(M^+ \to \xi^+_a \nu_a)} = \frac{|U_{ai}|^2 \rho(\delta^M_a, \delta^M_i)}{\rho(\delta^M_a, \delta^M_i)} \left(1 - \frac{\delta^M_i}{\delta^M_a}\right)^2,$$

(1)

where

$$\rho(x, y) = f_m(x, y) \lambda^{1/2}(1, x, y),$$

(2)

$$f_m(x, y) = x + y - (x - y)^2,$$

(3)

and

$$\delta^M_a = m^2_a/m^2_M, \quad \delta^M_i = m(\nu_i)^2/m^2_M.$$  

(4)

In eq. (2) we have displayed the kinematic rate factor $\rho$ in terms of a part $f_m \propto \lambda^{1/2}/(1, \delta^M_a, \delta^M_i)$ and another $\rho(\delta^M_a, \delta^M_i)$, $\propto$ to the two-body phase space factor. The well-known helicity effect acts to enhance $f_m$ as $\delta_i$ increases; for fixed $\delta_a, f_m$ increases from a minimum at $\delta_i = 0$ to a maximum at $\delta_i = \frac{1}{2} + \delta_a \pm 7$. The magnitude of this increase is indicated by the ratio

$$(f_m)_{max}/f_m(\delta_i, 0) = (2\delta^M_a + \frac{3}{2})/[\delta^M_a(1 - \delta^M_a)].$$

For $K^+ \to \mu^+ \nu_i$ this increase is a factor of eight; for $M^+ \to e^+ \nu_i$ it is approximately $(4\delta^M_a)^{-1}$ and hence quite dramatic, $2 \times 10^4$ for $\pi^+ \to e^+ \nu_i$ and $2 \times 10^5$ in the case of $K^+ \to e^+ \nu_i$. In the $M^+ \to \mu^+ \nu_i$ decays this effect offsets, and in the $M^+ \to e^+ \nu_i$ decays it completely overwhelms, the monotonous decrease of the phase space factor $\lambda^{1/2}/(1, \delta^M_a, \delta^M_i)$ until $m(\nu_i)$ reaches values rather near to the kinematic limits. Thus, for example, in the decays $\pi^+ \to \mu^+ \nu_i, \pi^+ \to e^+ \nu_i, K^+ \to \mu^+ \nu_i$, and $K^+ \to e^+ \nu_i$, the relative rate factors $\rho(\delta^M_a, \delta^M_i)/\rho(\delta^M_a, 0)$ reach maximal values of $1.00004, 1.11 \times 10^4, 4.13$, and $1.38 \times 10^5$ at $m(\nu_i)$ = 3.4, 80.5, 263, and 285 MeV, respectively.

Two important points follow from this analysis. First, a search for additional spectral lines in $M^+ \to \xi^+_a \nu_i$ is not inhibited by kinematic suppression until $m(\nu_i)$ reaches almost its phase space limit, as a consequence of the helicity enhancement effect together with the slow falloff of the two-body phase space factor; indeed, in the $M^+ \to e^+ \nu_i$ and $K^+ \to \mu^+ \nu_i$ decays there is actually very strong net kinematic enhancement up to quite large values of $m(\nu_i)$. This is to be contrasted with the case for three-body decays involving massive neutrinos; for example, in the decay $\mu \to \nu_e \nu_i$ (HSC $i$, LDC $j$) for the value $m(\nu_i)/m_\mu = 0.5$, the kinematic rate factor [5] is only $\approx 0.16$ of its value at $m(\nu_i) = 0$. Thus the above test is quite sensitive to neutrino masses throughout most of the kinematically allowed ranges for the various specific decays. Secondly, because of the monochromatic nature of the signal, the test can be applied on an event-by-event basis, in contrast to tests using three-body decays, in which one must search for deviations from a continuous momentum or energy distribution. Hence the test is capable of finding a very small signal or, alternately, of setting a commensurately stringent correlated upper bound of the form “If $m(\nu_i) \in (m_{min}, m_{M} - m_a)$ [where $m_{min}$ is the minimum value of $m(\nu_i)$] such that the HSC line $|p^{(f)}_{a}|$ is experimentally resolvable from the LDC line(s) $|p^{(f)}_{a}|_{LDC} \approx |p^0_{a}| = |p^0_{a}|(m(\nu_i) = 0)$, then $R_{a} < e\rho(\delta^M_a, \delta^M_i)$, where $\epsilon$ is determined by the statistics and sensitivity of the particular experiment. Equivalently, in the correlative form, the bound reads “If $R_{a} > e\rho(\delta^M_a, \delta^M_i)$, then there exists no $m(\nu_i) \in (m_{min}, m_{M} - m_a)$.”

Values of $m(\nu_i)$ and $|U_{ai}|$ sufficiently large to be detected by our tests do not violate any rigorous constraints. For example, the Cowls-McClelland bound [6] $\sum_{i=1}^{n} m(\nu_i^2) \lesssim 40$ eV applies only to effectively stable $\nu_e$, $\nu_\mu$, and $\nu_\tau$, the ratio $B(M^+ \to e^+ \nu_e)/B(M^+ \to \mu^+ \nu_\mu)$, $M = \pi, K$, does place upper bounds on the $|U_{ai}|, a \neq i$, as functions of $m(\nu_i)$. In evaluating these it is crucial to take into account the cuts which are used to define “$e^+ \nu_e$” and “$\mu^+ \nu_\mu$” events experimentally. Thus an HSC $M^+ \to e^+ \nu_e$ mode would not even be counted as part of the $M^+ \to e^+ \nu_e$ sample if $m(\nu_i)$ were sufficiently large that $|p^{(f)}_{e}|$ fell below the lower $|p^0_{e}|$ cut! Massive $\nu_i, i \in \{H\}$, will also cause $\nu$ oscillations, with wavelengths so short that any existing experimental constraints.

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Footnotes:

[6] The denominator of $R_{ai}$ is really $\Sigma \epsilon \{I_{L}\} |U_{ai}|^2$ but any LSC modes make a negligible contribution relative to the LDC mode(s). In a process such as $M^+ \to \xi^+_a \nu_a$ the denominator might include some HSC modes $i$ if $m(\nu_i) - 0.1$ keV but $m_a$. These, like the LSC modes, give a small contribution, and are suppressed in the notation.

[8] This maximum in $f_m$ is in the physical region if $m_a < m_M/4$, which condition is satisfied in $K\bar{\mu}_2, K\bar{e}_2$ and $\pi\bar{\mu}_2$ decay but not in $\pi\mu_2$ decay.

[8] For further bounds, see refs. [7,8]. Some of the bounds of ref. [8] rely on model-dependent astrophysical inputs.
would observe only an average effect. The present data on this type of oscillations is consistent with $|U_{\mu i}|$, $a = 1, 2$; HSC $i$, of order a few per cent.

The ultimate sensitivity of this test is limited by such factors as the e or $\mu$ energy loss in the target and by soft bremsstrahlung, and the resolution in $|p_a|$ or $E_a$ of the detector. Two possible background processes include $M^+ \to \ell^-_a \nu_\gamma$ and, for $M = K$, also $K^+ \to \pi^0 \ell^+_a \nu_j$ (LDC $j$). However, since their $|p_a|$ distributions are continuous and can be calculated, and since such events can be vetoed in an experiment with good photon detection capability, they should not constitute important backgrounds. Concerning the question of whether the test would be more advantageously applied to $M^+ \to e^+ \nu_j$ or $M^+ \to \mu^+ \nu_j$ decay, several comments are relevant. Since the helicity enhancement of the $i$th HSC mode is much greater, relative to the LDC mode(s), in the former case, the electron modes offer potentially greater sensitivity to small $|U_{\mu i}|^2$. Also, of course, if $m(\nu_i)$ is sufficiently large, the decay of an $M^+ = \pi^+$ or $K^+$ into $\mu^+ \nu_j$ may be kinematically forbidden while that into $e^+ \nu_j$ is still allowed. On the other hand, to the extent that lepton mixing is, like quark mixing, hierarchical, i.e. $|U_{\mu i}| < |U_{\tau i}|$, if $|a - i| > |b - j|$, one would expect that a given HSC mode $i$ would be more weakly coupled in $M^+ \to e^+ \nu_i$ than in $M^+ \to \mu^+ \nu_i$ decay. Perhaps most important, the rate for $M^+ \to e^+ \nu_j$ (LDC $j$) is much smaller than that for $M^+ \to \mu^+ \nu_j$, hence a search for HSC lines in $M^+ \to e^+ \nu_j$ decay is hindered by the fact that $\mu^-$e misidentification would produce spurious HSC $M^+ \to e^+ \nu_j$ events, since $|p_{\mu}(\text{LDC} j)| < |p_{e}(\text{LDC} j)|$.

Although no experiment has searched for additional spectral lines in $|p_a|$ or $E_a$, there is data to which we can apply our test. We begin with the SIN $\pi \mu_2$ experiment [2] and concentrate on the part of the data taken with the wider momentum acceptance (spectra #1–12 in ref. [2]). In this data $|p_{\mu}|_{\text{min}} / |p_{\mu}|_{0} \approx 0.98$ corresponding to $m(\nu_i)_{\text{max}} \approx 6.4$ MeV. The experimental appearance of the LDC line(s) at $|p_{\mu} |_{0}$ is a shoulder, with a tail extending below 0.98 $|p_{\mu} |_{0}$. An HSC line would thus appear as a second shoulder. From these spectra we can conclude that for $0.6 < m(\nu_i) < 6$ MeV, $R_{2i} \lesssim 0.05 \delta_\mu^2 / \rho(\delta_\nu^2, \delta_i^2)$. For the indicated range of $m(\nu_i)$ the RHS equals 0.05. (One may note a systematically high point at $|p_{\mu}(\text{LDC} j)| = 0.99 \times |p_{\mu} |_{0}$; however, this does not constitute a second shoulder and in any case is only a $\approx 1.5 \sigma$ effect.)

We next analyze the data from the CERN–Heidelberg $K_{\mu_2}$ experiment [4]. Here the momentum cuts for the $\mu$ and e events were $220 < |p_{\mu} |_{0} < 252$ MeV and $240 < |p_e|_{0} < 260$ MeV (where $|p_{\mu} |_{0} = 235.53$ and $|p_e|_{0} = 246.83$ MeV). The momentum spectra below 220 MeV were not shown. The peak shapes of the LDC $K_{\mu_2}$ and $K_{e_2}$ lines are sufficiently broad that one can infer only rather rough upper bounds on additional HSC modes. Moreover, the $K_{e_2}$ sample is limited by relatively small statistics. Concerning HSC $K_{\mu_2}$ modes, we conclude that for $230 > |p_{\mu}(\text{LDC})| > 220$ MeV $[70 < m(\nu_e) < 118$ MeV],

$$R_{2i} \lesssim 0.3 \delta_\mu^2 K (1 - \delta_\mu^2)^2 / \rho(\delta_K, \delta_i).$$

The RHS varies from 0.3 (0.69) to 0.3 (0.46) as $m(\nu_e)$ increases from 70 to 118 MeV. The corresponding limits for the HSC $K_{e_2}$ modes are: for $240 > |p_{e}(\text{LDC})| > 220$ MeV $[82 < m(\nu_e) < 163$ MeV],

$$R_{1i} \lesssim 0.25 \delta_\mu^2 K (1 - \delta_\mu^2)^2 / \rho(\delta_K, \delta_i).$$

Here the RHS varies from 0.25 (4.1 $\times 10^{-5}$) to 0.25 (1.2 $\times 10^{-5}$) as $m(\nu_e)$ increases from 82 to 163 MeV. Although we have used the events with $220 < |p_{\mu,e} |_{0} < 230$ MeV for our bounds, we record the warning that [4] "below 230 MeV the spectrum shape is affected by an imprecise momentum cut in the early stages of the analysis".

Finally we consider the Columbia experiment [3] which determined $B(\pi^+ \to e^+ \nu_e)/B(\pi^+ \to \mu^+ \nu_\mu)$ using timing techniques and energy measurement of the direct and $\mu$-decay electrons in a NaI(Tl) crystal. Since neither $|p_{\mu} |_{0}$ nor $E_\mu$ were measured, it is not possible to search for HSC $\pi^+ \to e^+ \nu_j$ modes in this data. Moreover, because of the much larger yield of e's from the $\pi \to \mu \to e$ decay chain, it is not practical to search for HSC $\pi^+ \to e^+ \nu_j$ modes with $E_\mu \approx 53$ MeV $[m(\nu_e) \gtrsim 68$ MeV]. The $\pi \to e$ spectrum does show some deviations from a one-peak fit; however, these were attributed to a periodic analyzer distortion (which also affected the $\pi \to \mu \to e$ sample). It would be valuable to re-analyze this data with this distortion removed.

In addition to $|p_{\mu}(\text{LDC})|$ or $E_\mu$ there is another quantity which is experimentally measurable, namely the longitudinal polarization $P_{\mu}^{(0)}(a = 2, \text{i.e. } \mu)$ of the muon in the $M^+ \to \mu^+ \nu_j$ decay modes. (This is not feasible without rescattering the electron in the $M^+ \to e^+ \nu_j$ decays.) With the standard weak couplings, the longitudinal polarization of $\ell^+_\mu$ is given by
In the cases where \( p_i \) is experimentally accessible, it decreases noticeably from its \( m(\nu_i) = 0 \) value of 1 for values of \( m(\nu_i) \) which are substantially below their kinematic limits. Thus, an extension of the test which, however, does assume V–A couplings, would be to check that the measured value of \( p_i \) agreed with the value inferred from \( m(\nu_i) \) extracted from the primary \( p_i \) or \( E_i \) measurement. Unfortunately, we know of no data to which we can apply this combined test.

The second type of process to which our general test for neutrino masses can be applied is nuclear decay. Since this is a three-body decay, it does not provide the monochromatic signal and corresponding sensitivity to small \( |u_{ai}| \) of the first test. However, it has the advantages of greater statistics and sensitivity to \( m(\nu_i) \) in the range from \( \approx 0.1 \) keV to several MeV; the \( M + \rightarrow + a v_i \) test could not detect to \( m(\nu_i) \leq m_\mu \). The \( E_\mu \) spectrum has been used to obtain the best upper bound on \( \langle m(\nu_e) \rangle \), by searching for a deviation in the Kurie plot, \( |N/(E_\mu E_\nu)|^{1/2} \) as a function of \( E_\mu \), as \( E_\mu \rightarrow (E_{\mu \text{max}}) \). An analysis of the decay \( 3^H \rightarrow 3^He + e^- + \nu_e \) yielded the best upper limit quoted as \( \langle m(\nu_e) \rangle < 35 \) eV (90\% CL) \[10\]. The precise meaning is that \( m(\nu_i) < 35 \) eV for all LDC modes \( i \) occurring in this \( \beta \) decay. In fact, however, previous discussions do not seem to have recognized that, as a corollary of our beginning observation, the early falloff near \( (E_{\mu \text{max}}) \) in a Kurie plot is not the only signature of massive neutrinos. Rather, a Kurie plot would in general consist of \( k' \) components due to the separate decays \( (Z_1, A_1) \rightarrow (Z_1 \pm 1, A_1) + e^+ + (\bar{\nu}_e) \). Of these, a subset \( i \in \{i_L\} \) would appear indistinguishable from a single transition, with net coupling strength
\[
\sum_{i \in \{i_L\}} |U_{Li}|^2 \approx \sum_{\text{LDC} i} |U_{Li}|^2 ,
\]
and a linear Kurie plot having no noticeable early endpoint falloff. The set \( \{i_L\} \) must contain at least one member, given the bound \[10\] on \( \langle m(\nu_e) \rangle \). The rest of the \( k' \) components would be HSC modes \( i \in \{i_H\} \), as allowed by phase space.

\[\delta = (\delta_\alpha - \delta_i) \lambda^{1/2} (1, \delta_\alpha, \delta_i) \]
\[
(\delta_\alpha + \delta_i) - (\delta_\alpha - \delta_i)^2 \] .

(5)

Our indirect bounds on modes which are SC in \( M + \rightarrow + a v_i \) and nuclear \( \beta \) decay can of course be supplemented by direct limits from decays where the same modes are DC. For example \( v_3 \) could, if allowed by phase space, be an SC mode in the former decays. In contrast, it is presumably (necessarily, if \( n = 3 \)) a DC mode in \( \tau \) decay. Thus, the decays \( \tau \rightarrow \nu_\tau \bar{\nu}_e \) and \( \tau \rightarrow \nu_\tau \nu_\mu \) yield the bounds, \( \langle m(\nu_v) \rangle \) [in particular, \( m(\nu_3) \) < 250 MeV (90\% CL) \[12\] and < 250 MeV \[13\], respectively. to the earlier attitude that only the well-measured \( \beta \) decay with the smallest \( Q \) value, \( 3^H(\beta^-)_{3^He} \), was useful in a search for \( \langle m(\nu_v) \rangle \neq 0 \), it is clear that in principle every \( \beta \) decay can be used. The characteristic signature of the \( i \)th HSC mode is a kink in the Kurie plot at its endpoint energy
\[
(E_{i \text{max}}) = \frac{(M_1^2 + m_\nu^2 - [M_2 + m(\nu_i)]^2)/(2M_1)}{2} .
\]

where \( M_1, 2 \) are the initial and final nuclear masses, together with the small incremental addition which it contributes for \( E_\mu < (E_{i \text{max}}) \). From the position of the \( i \)th kink one can determine \( m(\nu_i) \). Next, one can determine \( R_{i i} \) by measuring the increment due to this \( i \)th HSC mode and taking into account that
\[
dN/dE_\mu \propto |p_\mu |E_\mu E_\mu |^{1/2} (M_2, q_1^2, m(\nu_i)^2)/q_1 E_\mu ,
\]
where
\[
q_1^2 = M_2^2 + m_\mu^2 - 2M_1 E_\mu ,
\]
rather than
\[
dN/dE_\mu \propto |p_\mu |E_\mu E_\mu \text{LDC } \nu [(E_{i \text{max}} - E_\mu)]/q_1 E_\mu ,
\]
where
\[
(E_{i \text{max}}) = \left[ M_1^2 + m_\nu^2 - M_2^2 \right]/(2M_1) .
\]

We have applied this test to the Kurie plots for a large number of \( \beta \) decays, including all superallowed decays and a number of forbidden decays for which the shape correction factors are well known\[9\]. A consistency requirement is that any candidate HSC mode must appear at a constant energy increment below \( (E_{i \text{max}}) \) in every transition with sufficiently large \( Q \) value. Concerning possible backgrounds, one can distinguish between a true HSC mode and a lower energy branch (LEB) in a branched \( \beta \) spectrum because (a) the former will show curvature, especially near its endpoint, whereas the latter will be linear; (b) an LEB can
often be identified as such from a knowledge of the level structure of the final nucleus; (c) an LEB will fail the consistency check mentioned above, especially in comparison with unbranched $\beta$ spectra. However, Kurie plots often deviate upward from linearity at low $E_e$ because of finite source thickness ($e^+$ energy loss in the source), as well as various nuclear-dependent factors. Being cognizant of these effects, we have found no definitely affirmative evidence for HSC modes in nuclear $\beta$ decay and obtain a rough limit that for $m(\nu_i) \leq (0.1$ keV, $\approx 3$ MeV), $R_{\beta} \approx 0.1$. Further details will be reported elsewhere.

Other decays to which our general class of tests could be applied include $\mu$, $K_{e3}$, and hyperon decays. However, these are not expected to provide as sensitive probes for HSC modes.

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References